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Degree splitting of some *I*-cordial graphs

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Abstract- An *I*-cordial labeling of a graph G (V, E) is an injective map f from V to $\left[-\frac{p}{2}..\frac{p}{2}\right]^*$ or $\left[-\left\lfloor\frac{p}{2}\right\rfloor..\left\lfloor\frac{p}{2}\right\rfloor\right]$ as p is even or odd, which induces an edge labeling f^*: E → {0, 1} defined by $f^*(uv) = 1$ if f(u) + f(v) > 0 and $f^*(uv) = 0$ otherwise, such that the number of edges labeled with1and the number of edges labeled with 0 differ atmost by 1. If a graph has *I*-cordial labeling, then it is called ***I*-cordial graph**. In this paper, we prove that degree splitting of some well known graphs is *I*-cordial and some regular graph are not *I*-cordial.
Notation: Here [-x..x] = {t/t is an integer and |t| ≤ x} and $[-x..x]^* = [-x..x] - \{0\}$.

Keywords: Cordial labeling; *I*-cordial labeling.
AMS Mathematical Subject Classification (2010): 05C78

1. INTRODUCTION

By a graph we mean a finite undirected graph without loops and multiple edges. For terms not defined here we refer to Harary **[4].**

An *I*-cordial labeling of a graph G (V, E) is an injective map f from V to $\left[-\frac{p}{2}..\frac{p}{2}\right]^*$ or $\left[-\left\lfloor\frac{p}{2}\right\rfloor..\left\lfloor\frac{p}{2}\right\rfloor\right]$ as p is even or odd, which induces an edge labeling f^*: E → {0, 1} defined by $f^*(uv) = 1$ if f(u) + f(v) > 0 and $f^*(uv) = 0$ otherwise such that the number of edges labeled with 1 and the number of edges labeled with 0 differ at most by 1. If a graph has *I*-cordial labeling, then it is called ***I*-cordial graph**. The concept of cordial graph originated from I.Cahit [1,2] in 1987 as a weaker version of graceful and harmonious graphs and was based on {0,1} binary labeling of vertices.

Let f: V → {0, 1} be a mapping that induces an edge labeling $\overline{f}$: E → {0, 1} defined by $\overline{f}(uv) = |f(u) - f(v)|$. Cahit called such a labeling cordial if the following condition is satisfied: $|v_f(0) - v_f(1)| \le 1$and $|e_f(0) - e_f(1)| \le 1$,where $v_f(i)$ and $e_f(i)$, i = 0, 1 are the number of vertices and edges of G respectively with label i (under f and $\overline{f}$ respectively). A graph G is called cordial if it admits cordial labeling.

In **[3],** Cahit showed that (i) every tree is cordial (ii) K_n is cordial if and only if n ≤ 3 (iii) $K_{r,s}$ is cordial for all r and s (iv) the wheel W_n is cordial if and only if n ≡ 3(mod4) (v) C_n is cordial if and only if n ≢2(mod4) (vi) an Eulerian graph is not cordial if its size is congruent to 2 modulo 4. Few results can be seen in **[1,2]**.

In **[5]** we introduced the concept of *I*-cordial and proved that we prove that some standard graphs such as cycle C_n, PathP_n, Friendship F_n, Helm graph H_n, Closed graph CH_n, Double Fan DF_n, n ≥ 2, are *I*-cordial; Wheel W_n and Fan graph f_n are *I*-cordial if and only if n is even and complete graph K_p is not *I*-cordial. In **[6]** we proved that $B_{m,n}$, $S'(B_{n,n})$, $D_2(B_{m,n})$

are *I*-cordial; $K_{n,n}$ is *I*-cordial only if n is even ; $K_{m,n}$ is *I*-cordial only if m or n is even and $B_{n,n}^2$ is not *I*-cordial

The concept of degree splitting graph was introduced by R.Ponraj and S.Somasundaram **[7]**.

Definition: Let G = (V, E) be a graph with $V = S_1 \cup S_2 \cup \ldots \cup S_t \cup T$ where each S_i is a set of vertices having at least two vertices and having the same degree and $T = V - \bigcup_{i=1}^{t} S_i$. The degree splitting of graph G is denoted by DS(G) is obtained from G by adding vertices $w_1, w_2, \ldots, w_t$ and joining w_i to each vertex of S_i $(1 \le i \le t)$.

In this paper we prove that degree splitting of path, Wheel, gear graphs are I-cordial and $DS(K_2 + rK_1)$ is *I*-cordial if and only if r is odd, and degree splitting of regular graph is not *I*-cordial.

Theorem 1.1 $DS(P_n)$ is *I*-cordial.

Proof. Let P_n be the path $u_1, u_2, \ldots u_n$. Let $V(DS(P_n)) = \{u_1, u_2, \ldots u_n\} \cup \{u, v\}$ and $E(DS(P_n)) = \{uu_i ; 2 \le i \le n-1\} \cup \{uu_1, vu_n\}$. Here p = n + 2 and q = 2n – 1. We consider two cases:

CASE 1. n is even.

Let n = 2m. Then p = 2m + 2.

We Define $f : V(DS(P_n)) \rightarrow [-(m + 1) \,.\,.\, (m + 1)]^*$ by

$$f(u) = 1,$$
$$f(v) = -1,$$
$$f(u_i) = i + 1;\ 1 \le i \le m \text{ and}$$
$$f(u_{m+i}) = -(i + 1);\ 1 \le i \le m.$$

Then

$$f^*(u_iu_{i+1}) = \begin{cases} > 0, \text{for } i = 1, 2, 3, \ \ldots, m, \\ < 0, \text{for } i = m+1, \ \ldots, 2m-1. \end{cases}$$

Since f(u) = 1,

$$f^*(uu_i) = \begin{cases} > 0, \text{for } i = 2, 3, \ \ldots, m, \\ < 0, \text{for } i = m+1, \ \ldots, 2m-1. \end{cases}$$

Since f(v) = -1, $f^*(vu_1) > 0$ and $f^*(vu_n) < 0$.

Hence, $e_f(0) = m + m - 1 + 1 = 2m$ and

$$e_f(1) = m - 1 + m - 1 + 1 = 2m - 1.$$

Thus, $|e_f(0) – e_f(1)| = 1$.

CASE 2. n is odd.

Let n = 2m + 1. Then p = 2m + 3.

We Define $f : V(DS(P_n)) \rightarrow [(m + 1) \,.\,.\, -(m + 1)]$ by

$$f(u) = 1,$$
$$f(v) = -1,$$
$$f(u_i) = i + 1;\ 1 \le i \le m,$$
$$f(u_{m+1}) = 0 \text{ and}$$
$$f(u_i) = -(i + 1);\ 1 \le i \le m.$$

Then $$f^*(u_iu_{i+1}) = \begin{cases} > 0, \text{for } i = 1, 2, 3, \ \ldots, m, \\ < 0, \text{for } i = m+1, \ \ldots, 2m. \end{cases}$$

Here m edges evenly share positive and negative integers.

Since f(u) = 1,

$$f^*(uu_i) = \begin{cases} > 0, \text{for } i = 2, 3, \ \ldots, m+1, \\ < 0, \text{for } i = m+2, \ \ldots, 2m. \end{cases}$$

Hence m edges receive positive labels and (m – 1) edges receive negative labels.

Since f(v) = -1, $f^*(vu_1) > 0$ and $f^*(vu_{2m+1}) < 0$, we have

$$e_f(1) = m + m + 1 = 2m \text{ and}$$
$$e_f(0) = m + m – 1 + 1 = 2m.$$

That is, $|e_f(0) – e_f(1)| = 1$.

Therefore from both the cases $|e_f(0) – e_f(1)| \le 1$.

Thus $DS(P_n)$ is *I*-cordial. ■

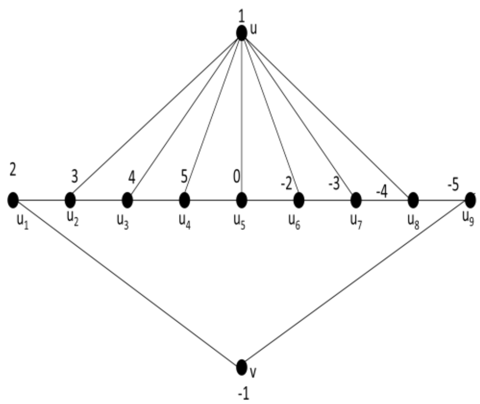

Fig. 1 *I*-cordial labeling of $DS(P_9)$

Theorem 1.2 Degree Splitting of any regular graph is not *I*-cordial.

Proof. Let G be a regular graph with n vertices and $d(G) = n - 1$. When a vertex is added, then $d(G) = n = p$, by Theorem 2.1, 2.2 in [5] G is not *I*-cordial. ■

Theorem 1.3 $DS(W_n)$, where $W_n = C_n + K_1$, $n > 3$ is *I*-cordial.

Proof. Let u be the apex vertex and $v_1, v_2, \ldots, v_n$ be the rim vertices of W_n.
Let $V(DS(W_n)) = V(W_n) \cup w$. Let us consider two cases:

CASE 1. n is even.

Let $n = 2m$. Then $p = 2m + 2$.

We define $f : V(DS(W_n)) \to [\ -(m + 1) \ . \ . \ (m + 1)]^*$ by,

$$f(u) = 1,$$
$$f(w) = -1,$$
$$f(v_i) = i + 1;\ 1 \le i \le m \text{ and}$$
$$f(v_{m+i}) = -(i + 1);\ 1\le i \le m.$$

Since $f(u) = 1$,

$$f^*(uv_i) = \begin{cases} > 0, \text{for } i = 1, 2, 3, \ \ . . ., m, \\ < 0, \text{for } i = m+1, \ \ . . ., 2m. \end{cases}$$

$$\text{Now, } f^*(v_iv_{i+1}) = \begin{cases} > 0, \text{for } i = 1, 2, 3, \ \ . . ., m \\ < 0, \text{for } i = m+1, \ \ . . ., 2m-1 \end{cases}$$

and $f^*(v_{2m}v_1) < 0$.

Since $f(w) = -1$,

$$f^*(wv_i) = \begin{cases} > 0, \text{for } i = 1, 2, 3, \ \ . . ., m, \\ < 0, \text{for } i = m+1, \ \ . . ., 2m. \end{cases}$$

Hence, $e_f(0) = m + m + m = 3m$ and

$$e_f(1) = m + m + m = 3m.$$

Thus, $|e_f(0) - e_f(1)| = 0$.

CASE 2. n is odd.

Let $n = 2m + 1$. Then $p = 2m + 3$.

We define $f : V(DS(W_n)) \to [(m + 1) \ . \ . \ -(m + 1)]$ by

$$f(u) = 1,$$
$$f(w) = -1,$$
$$f(v_i) = i + 1;\ 1 \le i \le m,$$
$$f(v_{m+1}) = 0 \text{ and}$$
$$f(v_{m+1+i}) = -(i + 1);\ 1 \le i \le m.$$

Since $f(u) = 1$,

$$f^*(uv_i) = \begin{cases} > 0, \text{for } i = 1, 2, 3, \ \ . . ., m+1, \\ < 0, \text{for } i = m+2, \ \ . . ., 2m+1. \end{cases}$$

$$\text{Now, } f^*(v_iv_{i+1}) = \begin{cases} > 0, \text{for } i = 1, 2, 3, \ \ . . ., m, \\ < 0, \text{for } i = m+1, \ \ . . ., 2m. \end{cases}$$

and $f^*(v_{2m+1}v_1) < 0$.

Since, $f(w) = -1$,

$$f^*(wv_i) = \begin{cases} > 0, \text{for } i = 1, 2, 3, \ \ . . ., m, \\ < 0, \text{for } i = m+1, \ \ . . ., 2m+1. \end{cases}$$

Hence, $e_f(0) = m + (m + 1) + m = 3m + 1$ and

$$e_f(1) = (m + 1) + m + m + 1 = 3m + 2.$$

Thus, $|e_f(0) - e_f(1)| = 1$.

From both the cases we get $|e_f(0) - e_f(1)| \le 1$.

Hence $DS(W_n)$, $n > 3$ is *I*-cordial.

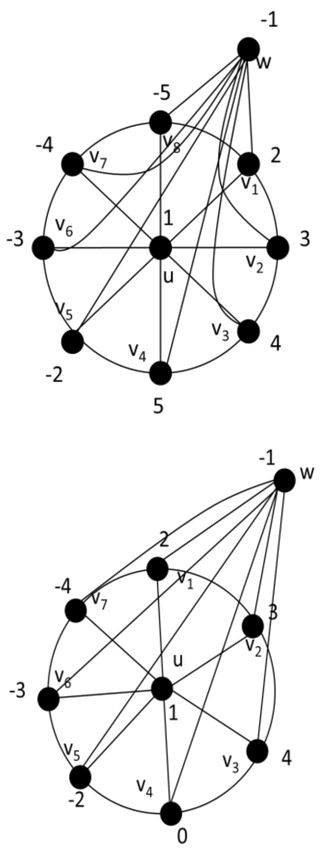

Fig. 2 *I*-cordial labeling of **(a)** DS(W_8) and **(b)** DS(W_7).

Theorem 1.4 DS(G_n), $n > 3$, where G_n is gear graph, is *I*-cordial.

Proof. Let u be the apex vertex and $v_1, v_1', v_2, v_2', \ldots, v_n, v_n'$ be the vertices of cycle. Let w_1, w_2 be the newly added vertices. Then $E(DS(G_n)) = \{uv_i, 1 \leq i \leq n\} \cup \{v_iv_i', 1 \leq i \leq n\} \cup \{v_n' v_1\} \cup \{w_1v_i, 1 \leq i \leq n\} \cup \{w_2v_i', 1 \leq i \leq n\}$.

We define $f : V \rightarrow [-\frac{p}{2} .. \frac{p}{2}]^*$ as

$f(u) = 0$,

$f(w_1) = 1$,

$f(w_2) = -1$ and

The vertices $v_1, v_1', v_2, v_2', \ldots, v_{\frac{n}{2}}, v_{\frac{n}{2}}'$ are labeled 2, 3, . . ., (n + 1), and the vertices $v_{\frac{n+1}{2}}, v_{\frac{n+1}{2}}', \ldots, v_n, v_n'$ with -2, -3, . . ., –(n + 1).

so that $f^*(v_iv_i') = \begin{cases} > 0, \text{for } i = 1, 2, 3, \ldots, \frac{n}{2}, \\ < 0, \text{for } i = \frac{n}{2}+1, \ldots, n. \end{cases}$

$$f^*(v_i'v_{i+1}) = \begin{cases} > 0, \text{for } i = 1, 2, 3, \ldots, \frac{n}{2}, \\ < 0, \text{for } i = \frac{n}{2}+1, \ldots, n-1. \end{cases}$$

and $f^*(v_n'v_1) < 0$.

Since $f(v) = 0$,

$$f^*(vv_i) = \begin{cases} > 0, \text{for } i = 1, 2, 3, \ldots, \frac{n}{2}, \\ < 0, \text{for } i = \frac{n}{2}+1, \ldots, n. \end{cases}$$

Since $f(w_1) = 1$,

$$f^*(w_1v_i) = \begin{cases} > 0, \text{for } i = 1, 2, 3, \ldots, \frac{n}{2}, \\ < 0, \text{for } i = \frac{n}{2}+1, \ldots, n. \end{cases}$$

Since $f(w_2) = -1$,

$$f^*(w_2v_i') = \begin{cases} > 0, \text{for } i = 1, 2, 3, \ldots, \frac{n}{2}, \\ < 0, \text{for } i = \frac{n}{2}+1, \ldots, n. \end{cases}$$

Here $e_f(0) = \frac{n}{2} + \frac{n}{2} - 1 + 1 + n + \frac{n}{2} = \frac{5n}{2}$ and

$$e_f(1) = \frac{n}{2} + \frac{n}{2} + n + \frac{n}{2} = \frac{5n}{2}$$

Therefore, $|e_f(0) - e_f(1)| \leq 1$. Hence, G_n, $n > 3$ is *I*-cordial.

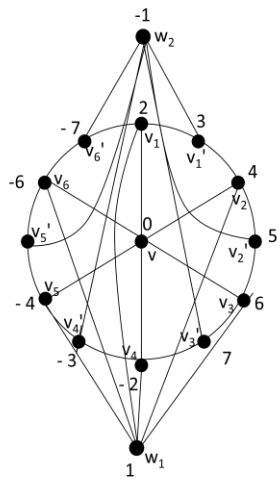

Fig. 3 DS(G_6) is *I*-cordial.

Theorem1. 5 DS(K_2 + rK_1) is *I*-cordial if and only if r is odd.

Proof. Let V(DS(K_2 + rK_1)) = {u, v, w_i, $1 \le i \le r$} ∪ {a, b} and E(DS(K_2 + rK_1)) = {uwi, $1 \le i \le r$} ∪ {vwi, $1 \le i \le r$} ∪ {au_1} ∪ {bw_i, $1 \le i \le r$}. Then the graph consists of (r + 4) vertices and 3r + 3 edges.

Let r =2m + 1.

We consider f : V → [-(m + 2) . . (m + 2)], defined by

f(a) = 2,

f(b) = -1,

f(u) = 0,

f(v) = 1,

f(w_{2m+1}) = -2 and

The other vertices w_1, w_2, . . ., w_{2m} is labeled as 3 , -3, . . ., 2m , -2m.

Since f(u) = 0,

$$f^*(uw_i) = \begin{cases} > 0, \text{ for } i = 1, 3, 5, \ \ldots, 2m - 1, \\ < 0, \text{ for } i = 2, 4, 6 \ \ldots, 2m, 2m+1. \end{cases}$$

Since, f(v) = 1, $f^*(uv) > 0$, $f^*(va) > 0$,

$$f^*(vw_i) = \begin{cases} > 0, \text{ for } i = 1, 3, 5, \ \ldots, m - 2, \\ < 0, \text{ for } i = 2, 4, 6 \ \ldots, 2m, 2m+1. \end{cases}$$

Also, f(b) = - 1, so that

$$f^*(bw_i) = \begin{cases} > 0, \text{ for } i = 1, 3, 5, \ \ldots, 2m - 1, \\ < 0, \text{ for } i = 2, 4, 6 \ \ldots, 2m, 2m+1. \end{cases}$$

Also, f(a) = 2, so that $f^*(au) > 0$.

Hence, $e_f(0)$ = (m + 1) + (m + 1) + m + 1 = 3m + 3 and

$e_f(1)$ = m + m + m + 3 = 3m + 3.

That is, $|e_f(0) - e_f(1)| = 0$.

Conversly, when n is even.

Assume G is cordial. Let f : V $\rightarrow [-\frac{p}{2} .. \frac{p}{2}]^*$ be an *I*-cordial labeling.

To avoid two equal labeling with different parity hitting two adjacent vertices, we shall assign them to a, b. Other pairs must occupy the adjacent vertices which would contradict the *I*-cordial condition. Therefore, DS(K_2 + rK_1) is not *I*-cordial.

Also $f(w_i) > 0$ for all i. Then the remaining four vertices will receive negative labels, such that at least one of them will neutralize $f(w_j)$ for some j, which is a contradiction to *I*-cordiality.

Hence DS(K_2 + rK_1) is not *I*-cordial.

Thus, DS(K_2 + rK_1) is *I*-cordial if and only if n is odd.

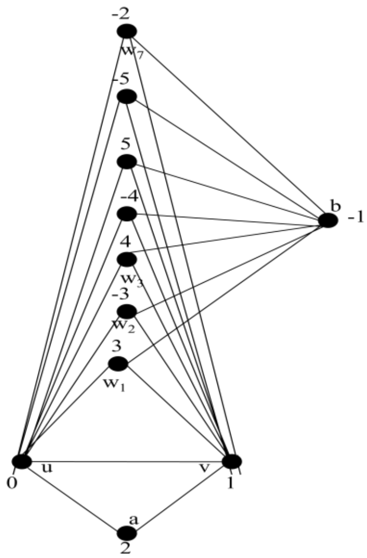

Fig. 4 DS (K_2 + $7K_1$) is *I*-cordial.

Theorem1. 6 Degree Splitting of n pages of pentagonal book is *I*-cordial.

Proof. Let G be the graph obtained by degree splitting of n pages of pentagonal book. Then V(G) = $\{u_i, v_i, w_i, 1 \le i \le n\} \cup \{u, v, a, b\}$ and E(G) = $\{uu_i\}_{i=1}^{n} \cup \{u_iw_i\}_{i=1}^{n} \cup \{vv_i\}_{i=1}^{n} \cup \{v_iw_i\}_{i=1}^{n} \cup \{u_ia\}_{i=1}^{n} \cup \{aw_i\}_{i=1}^{n} \cup \{av\}_{i=1}^{n} \cup \{uv\} \cup \{bu\} \cup \{bv\}$.Here p = 3n + 4 and q = 7n + 3. We consider two cases.

CASE 1. n is even.

Let n = 2m. Then p = 6m + 4.

We define $f : V \rightarrow [-(3m + 2) \,.\,.\, (3m + 2)]^*$ as,

$f(a) = 1$,

$f(b) = -2$,

$f(u) = 3m + 2$,

$f(v) = -1$,

$f(u_i) = i + 1;\ 1 \le i \le 2m$,

$f(v_i) = -(i + 2);\ 1 \le i \le 2m$,

$f(w_i) = (2m + 1 + i)\ ,\ 1 \le i \le m$ and

$f(w_i) = -(2m + 1 + i),\ 1 \le i \le m$.

Since f(a) = 1, so that $f^*(au_i) > 0$ for all i = 1, 2, . . ., 2m and

$$f^*(aw_i) = \begin{cases} > 0, \text{for } i = 1, 2, 3, \ \ . \ . \ ., m, \\ < 0, \text{for } i = m+1, \ \ . \ . \ ., 2m. \end{cases}$$

Now, $f^*(uu_i) > 0$ for i = 1, 2, . . ., 2m;

$f^*(vv_i) < 0$ for all i = 1, 2, . . ., 2m;

$$f^*(u_iw_i) = \begin{cases} > 0, \text{for } i = 1, 2, 3, \ \ . \ . \ ., m, \\ < 0, \text{for } i = m+1, \ \ . \ . \ ., 2m. \end{cases}$$

$$f^*(v_iw_i) = \begin{cases} > 0, \text{for } i = 1, 2, 3, \ \ . \ . \ ., m \text{ and} \\ < 0, \text{for } i = m+1, \ \ . \ . \ ., 2m. \end{cases}$$

Also, $f^*(uv) > 0$, $f^*(ub) > 0$ and $f^*(vb) < 0$.

Hence, $e_f(1)$ = 2m + 2m + m + m + m + 2 = 7m + 2 and

$e_f(0)$ = 2m + m + 2m + 1 + m + m = 7m + 1.

Hence, $|e_f(0) - e_f(1)| = 1$.

CASE 2. n is odd.

Let n = 2m + 1. Then p = 6m + 7.

We define $f : V \rightarrow [-(3m + 3) \,.\,.\, (3m + 3)]$ as,

$f(a) = -1$,

$f(b) = 1$,

$f(u) = 3m + 3$,

$f(v) = 0$,

$f(u_i) = i + 1;\ 1 \le i \le 2m + 1$,

$f(v_i) = -(i + 1);\ 1 \le i \le 2m + 1$,

$f(w_i) = 2(m + 1) + i;\ 1 \le i \le m$ and

$f(w_{m+i}) = -(2(m + 1) + i);\ 1 \le i \le m + 1$.

Since f(a) = -1, $f^*(au_i) > 0$, $f^*(av_i) < 0$ for all i = 1, 2, . . ., 2m + 1.

$$\text{Also, } f^*(aw_i) = \begin{cases} > 0, \text{for } i = 1, 2, 3, \ \ . \ . \ ., m, \\ < 0, \text{for } i = m+1, \ \ . \ . \ ., 2m+1. \end{cases}$$

Hence, 3m + 1 edges receive positive label and 3m + 2 edges receive negative labels.

Now, $f^*(uu_i) > 0$, $f^*(vv_i) < 0$ for all i = 1, 2, . . ., 2m + 1. $f^*(u_iw_i) > 0$ and $f^*(v_iw_i) < 0$ for all

$i = m + 1, \ldots, 2m + 1$. Hence, $m + 2$ edges receive positive labels and $m + 4$ edges receive negative labels.

Here $e_f(0) = 2m + 1 + 2m + 1 + m + 3 + m + m = 7m + 5$ and

$e_f(1) = 2m + 1 + m + 1 + 2m + 1 + m + 1 + m + 1 = 7m + 5$.

Hence, $|e_f(0) - e_f(1)| = 0$.

Thus, degree splitting of n pages of pentagonal book is *I*-cordial. ■

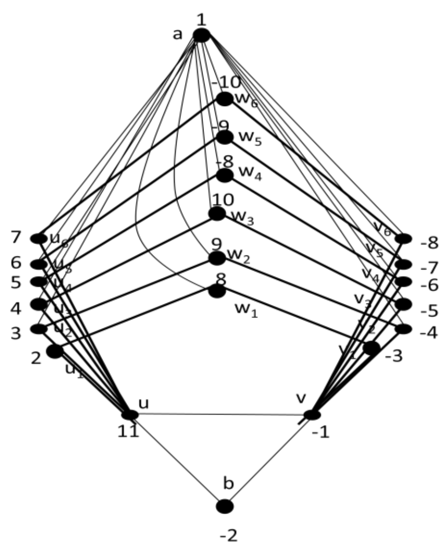

Fig. 4 *I*-cordial labeling of degree splitting of 6pages of pentagonal book

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